

Whose Knowledge?

N. David Mermin
Laboratory of Atomic and Solid State Physics
Cornell University, Ithaca, NY 14853-2501

(To appear in *Quantum (Un)speakables: Essays in Commemoration of John S. Bell*, eds. Reinhold Bertlmann and Anton Zeilinger, Springer Verlag, late 2001.)

Sir Rudolph Peierls, in a reply to John Bell's last critique of the state of our understanding of quantum mechanics, maintained that it is easy to give an acceptable account of the physical significance of the quantum theory. The key is to recognize that all the density matrix characterizing a physical system ever represents is *knowledge* about that system. In answer to Bell's implicit rejoinder "*Whose knowledge?*" Peierls offered two simple consistency conditions that must be satisfied by density matrices that convey the knowledge different people might have about one and the same physical system: their density matrices must commute and must have a non-zero product. I describe a simple counterexample to his first condition, but show that his second condition, which holds trivially if the first does, continues to be valid in its absence. It is an open question whether any other conditions must be imposed.

I met John Bell two and a half times. The two times, both memorable for me, were in what turned out to be in the last year of his life. The first was at a summer school in Erice in August, 1989, where he delivered what came close to being the most spell-binding lecture I have ever heard. (The only competitors are Richard Feynman's 1965 Messenger lectures at Cornell.) The text was subsequently published in a much quoted article in *Physics World*, "Against Measurement" [1]. The article conveys his brilliance and his wit, but not, of course, the music of his voice and the fire of his beard and hair.

The other moments I remember from our first meeting are Bell's polite but evidently skeptical response to my telling him that I had long ago written a book on relativity [2] that took the view he advocated several years later in "How to teach special relativity" [3]. Later I was pleased to get a letter saying that he had looked up my book and agreed that I did indeed do it right.

And I remember a reception at which there suddenly rang out over the noise of the crowd a cry of "Don't be a sissy!" It was John Bell, encouraging a younger physicist not to let the scope of his speculative investigations be overly constrained by the wisdom of his elders.

Our second meeting was for a week in Amherst in the summer of 1990. George Greenstein and Arthur Zajonc had invited about a dozen people, among them John and Mary Bell, Kurt Gottfried, Viki Weisskopf, Phillip Pearle, Tony Leggett, Danny Greenberger, Mike Horne, Anton Zeilinger, and Abner Shimony, to spend a week in a fraternity house chatting about the nature of quantum mechanics. No prepared talks, no schedule, no proceedings. Just wonderful conversations.

Shortly before the meeting I had written John — this was at the very end of the epoch in which one communicated at a leisurely pace on paper — to tell him about a simplified version I had constructed of a little known argument of Greenberger, Horne, and Zeilinger. I got back a wonderful reply, which I have searched for in my files unsuccessfully ever since he died. He said that the argument “filled me with admiration”, and he wrote down a relevant three-particle Bell inequality, which inspired me to find a stronger one [4]. (Somewhere, lost in my filing cabinet, is proof that the very first Bell inequality for “Bell’s Theorem without Inequalities” was Bell’s Bell inequality.) There were lots of entertaining discussions and arguments in Amherst. It was far and away the finest conference I have ever been to.

I also have evidence that over a third of a century ago we had a half meeting. The evidence is in the form of a conference photograph in my office taken in Birmingham England in the summer of 1967 at a meeting to celebrate the 60th birthday of Rudi Peierls. Amongst 100 white-shirted necktied gentlemen is a figure in the back row toward the left in a plaid shirt without a tie. He sports one of the only two beards in the photograph and is identified only as Bell; on the right in the rear is a more conventional looking youth identified as Mermin.

This Bell looks like an early version of the John Bell I later knew, and since the even younger John Bell spent a year in Birmingham in 1953-4 (where I spent two years in 1961-3), it seems clear that the Bell in the photo was indeed John. So I could have met John Bell almost a quarter of a century before I actually did meet him. It is probable that I was even introduced to him at that time, quite oblivious of the fact that I was meeting the discoverer of what would come to be called Bell’s theorem, just as it was quietly tiptoeing onto the intellectual scene.

Our virtual meeting through Peierls is relevant to my contribution to this volume, because what I have to say was inspired by another, more recent, link between Bell and Peierls. After John’s Erice lecture appeared in *Physics World*, but too late, alas, for him to read and reply to it, there appeared something between a commentary and a rebuttal by Peierls [5], by then, I calculate, a sprightly 84, which raised for me a puzzle which to this day I have not been able to answer to my satisfaction.

Peierls agreed with Bell that we should have “a clearly formulated presentation of the physical significance of the [quantum] theory without relying on ill-defined concepts” and

that “I do not know of any textbook which explains these matters to my satisfaction.” On the other hand, he added, “I do not agree with John Bell that these problems are very difficult. I think it is easy to give an acceptable account.” According to Peierls all it boils down to is this:

In my view the most fundamental statement of quantum mechanics is that the wavefunction, or, more generally the density matrix, represents our *knowledge* of the system we are trying to describe.

Interestingly, *knowledge* is not on Bell’s now famous list of

words which, however legitimate and necessary in application, have no place in a *formulation* with any pretension to physical precision.

But “information” is on the proscribed list, the charge against it being:

Information? Whose information? Information about what?

I believe we can take “information” and “knowledge” to be synonymous in this context.

Until quite recently I was entirely on Bell’s side on the matter of knowledge-information. But then I fell into bad company. I started hanging out with the quantum computation crowd, for many of whom quantum mechanics is self-evidently and unproblematically all about information. I digress to remark that among the tragedies of John Bell’s early death is his missing the quantum computation revolution of the 1990’s. It has been widely regretted that death deprived us of Einstein’s reaction to Bell’s theorem; another great loss is never to know what Bell would have had to say about Peter Shor’s factoring algorithm or Lov Grover’s search algorithm, and the novel view of quantum mechanics that emerges when you take the theory seriously as a source of impossibly fast algorithms for the processing of knowledge-information.

Partly from my associations with quantum computer scientists and partly from endless debates with constructivist sociologists of science, I have come to feel that “Information about *what?*” is a fundamentally metaphysical question that ought not to distract tough-minded physicists. There is no way to settle a dispute over whether the information is about something objective, or is merely information about other information. Ultimately it is a matter of taste, and, like many matters of taste, capable of arousing strong emotions, but in the end not really very interesting.

On the other hand “*Whose* information?” raises serious practical questions that simply have to be addressed. For while information, in the form of sense perceptions, is all any of us have direct access to, we have developed powerful ways to exchange with each other some of the content of our own private information. It is entirely legitimate and unambiguous to raise the question of what constraints, if any my possession of my knowledge imposes on the knowledge that you can possess, and vice versa.

Peierls was surely aware of Bell’s double-barreled shot at information-knowledge and immediately after declaring that it’s all about knowledge, he promised to “return later to the question ‘whose knowledge’”. I infer that like the very recent me, he did not find it necessary to deal with the question “knowledge about what?” But towards the end of his article he dealt briskly and efficiently with “whose knowledge?”:

[Density matrices] may differ, as the nature and amount of knowledge may differ. People may have observed the system by different methods, with more or less accuracy; they may have seen part of the results of another physicist. However, there are limitations to the extent to which their knowledge may differ. This is imposed by the uncertainty principle. For example if one observer has knowledge of S_z of our Stern-Gerlach atom, another may not know S_x , since measurement of S_x would have destroyed the other person’s knowledge of S_z , and vice versa. This limitation can be compactly and conveniently expressed by the condition that the density matrices used by the two observers must commute with each other.

In a chapter in *More Surprises in Theoretical Physics* to which he referred the reader for further details, Peierls added a second condition, which is easy to understand if the density matrices do indeed commute [6]:

At the same time, the two observers should not contradict each other. This means the product of the two density matrices should not be zero.

Peierls’ rationale for his first condition has a highly plausible ring to it. After all, if Alice *knows* that a spin- $\frac{1}{2}$ particle is in an eigenstate of S_z , then absolutely no information can be available to anybody about any components of its spin orthogonal to the z -direction. A measurement of any spin component in the x - y plane is as likely to come out up as down. Mathematically this requires that nobody’s density matrix can have off-diagonal elements in the S_z representation — i.e. everybody’s density matrix must commute with the pure-state density matrix of Alice.

Things become a little less clear if Alice’s knowledge is not characterized by a pure-state density matrix, and considerably less clear if we are dealing with more than a two-state system. For some time I struggled to find a general way to understand Peierls’ first criterion. I stopped struggling when Chris Fuchs pointed out to me that it was clearly incorrect, even for two-state systems and even when Alice’s density matrix described a pure state. I give here a slightly embellished version of Fuchs’ simple counterexample, which makes explicit how a disparity in knowledge can arise that leads Alice and Bob to describe one and the same physical system with non-commuting density matrices.

Consider a pair of qubits (spins- $\frac{1}{2}$, if you prefer, 0 being \uparrow along \mathbf{z} and 1 being \downarrow) in the state

$$|\Psi\rangle = \cos\theta|0\rangle|0\rangle + \sin\theta|1\rangle|1\rangle. \quad (1)$$

with $0 < \theta < \pi/4$. After the state has been prepared the qubits cease to interact. To emphasize the absence of subsequent interaction the qubit on the right (hereafter called the right qubit) can be carried far away from the other (hereafter called the left qubit). Anybody who knows how the two-qubit state was prepared — and, in particular, Alice and Bob, who both know — will agree that although the two-qubit system is in a pure state, each separate qubit, and in particular the left qubit, has no pure state of its own but is characterized, at the most fundamental level, by the mixed-state density matrix

$$\rho = \text{Tr}_r |\Psi\rangle\langle\Psi| = \cos^2\theta |0\rangle\langle 0| + \sin^2\theta |1\rangle\langle 1|, \quad (2)$$

where Tr_r is the partial trace over the degrees of freedom of the right qubit.

Suppose Alice now goes to the right qubit and secretly measures it in the computational basis. She does not report to Bob the result of her measurement or even whether she has measured it at all. Since the right qubit is far away and does not interact with the left qubit and since Alice communicates no information whatever to Bob, regardless of what she has or has not done to the right qubit, Bob must surely continue to describe the left qubit with the same density matrix (2). Alice, on the other hand, knowing the strong correlations present in the state (1), is able after measuring the right qubit to update the density matrix she uses to express her knowledge about the qubit on the left to one of the pure-state density matrices $|0\rangle\langle 0|$ or $|1\rangle\langle 1|$, depending on whether the result of her measurement on the right is 0 or 1. Since either of these commutes with Bob's density matrix (2), Peierls' first condition is confirmed.

Suppose, however, that prior to making her measurement Alice secretly applies to the right qubit the unitary Hadmard transformation,

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad |1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \quad (3)$$

This transforms the two-qubit state into

$$\begin{aligned} |\Phi\rangle &= \frac{1}{\sqrt{2}}(\cos\theta |0\rangle(|0\rangle + |1\rangle) + \sin\theta |1\rangle(|0\rangle - |1\rangle)) \\ &= \frac{1}{\sqrt{2}}((\cos\theta |0\rangle + \sin\theta |1\rangle)|0\rangle + (\cos\theta |0\rangle - \sin\theta |1\rangle)|1\rangle). \end{aligned} \quad (4)$$

Now, depending on whether Alice gets 0 or 1 from her measurement of the right qubit, she will assign the left qubit one or the other of the pure-state density matrices

$$\begin{aligned} \rho_a &= (\cos\theta |0\rangle \pm \sin\theta |1\rangle)(\cos\theta \langle 0| \pm \sin\theta \langle 1|) \\ &= \cos^2\theta |0\rangle\langle 0| + \sin^2\theta |1\rangle\langle 1| \pm \cos\theta \sin\theta (|0\rangle\langle 1| + |1\rangle\langle 0|). \end{aligned} \quad (5)$$

But Bob, who does not know what, if anything Alice did to the right qubit, must continue to use (2) for his density matrix ρ_b describing the left qubit. We therefore have

$$[\rho_b, \rho_a] = \pm \frac{1}{4} \sin 4\theta (|0\rangle\langle 1| - |1\rangle\langle 0|). \quad (6)$$

This commutator is non-zero for $0 < \theta < \pi/4$ regardless of the result of Alice's measurement. Peierls' criterion fails to hold.

Having lived and worked for two years in Peierls' Birmingham department, where I came to admire him immensely, I am adding to my list of reactions I wish death had not snatched from us Peierls' response to Fuchs' counterexample. All I know for sure, is that he would not have attempted to escape by maintaining that Alice's action on the right qubit had somehow altered the objective condition of the left qubit. He would have agreed that whatever she did to and learned about the qubit on the right altered only her *knowledge* of the faraway qubit on the left. Since Alice communicates none of this updated knowledge to Bob, there is no way for him to update his density matrix as a result of her actions. But since she now *knows* that the density matrix for the left qubit is a particular pure state density matrix, she is surely entitled to use it as the appropriate expression of her current knowledge.

Here is what seems to go wrong with Peierls' justification for his first condition. Depending on the result of her measurement on the right qubit, Alice's knowledge of the left qubit is indeed encapsulated in one of the two pure states

$$|\psi\rangle_+ = \cos \theta |0\rangle + \sin \theta |1\rangle \quad (7)$$

or

$$|\psi\rangle_- = \cos \theta |0\rangle - \sin \theta |1\rangle. \quad (8)$$

In spin language she therefore either knows that the left qubit is in an eigenstate of the spin along the direction $\mathbf{n}_+ = \cos 2\theta \mathbf{z} + \sin 2\theta \mathbf{x}$ or she knows that it is in an eigenstate of the spin along the direction $\mathbf{n}_- = \cos 2\theta \mathbf{z} - \sin 2\theta \mathbf{x}$. As Peierls noted, it is inconsistent with Alice's knowledge for Bob to know anything about the spin along any direction orthogonal to \mathbf{n}_+ in the former case, or orthogonal to \mathbf{n}_- in the latter case.

But Bob does not know which case is the case. Indeed, he does not even know whether Alice knows which case is the case and therefore (to get slightly metaphysical) whether there *is* a case which is the case. Should Alice happen to know that the state of the left qubit is $|\psi\rangle_+$, then it is not incompatible with Alice's knowledge for Bob to be able to make more than a completely random guess about the result of measuring a spin component in the plane perpendicular to \mathbf{n}_- , and vice-versa. The only direction along which Bob cannot improve on a random guess, regardless of whether $|\psi\rangle_+$ or $|\psi\rangle_-$ encapsulates Alice's knowledge of the left qubit, is the direction \mathbf{y} orthogonal to both \mathbf{n}_+

and \mathbf{n}_- . And Bob’s density matrix (2) does indeed give equal probabilities for his finding either value of S_y . That this state of affairs holds whatever unitary transformation Alice applies to the right qubit before she measures it in the computational basis, is shown in the Appendix below. So Bob gets away with violating Peierls’ constraint on what he can know because he does not know what knowledge Alice knows he cannot have, and therefore must continue to use a density matrix that embodies knowledge that it *might* be possible for him to have.

I do not believe that this counterexample is contrived or artificial. The initial density matrix (2) that both Alice and Bob assign to the left qubit is required by basic principles of quantum mechanics and the fact that what each of them knows about the two-qubit system is given by the pure state (1). Alice’s subsequent acquisition of information enabling her to refine her density matrix down to one or the other of the two choices of pure-state density matrix (5) cannot alter Bob’s assignment of density matrix, because the actions she takes to make the refinement are taken far from the left qubit. There is therefore no material alteration of the left qubit that Bob is, in some way, willfully ignoring. Nor can there be any change in Bob’s *knowledge* of the left qubit, since neither the nature of Alice’s actions nor the outcomes of those actions are communicated to Bob.

So to appropriate a remark John Bell once made about the Einstein-Podolsky-Rosen argument [7], Peierls’ first condition “doesn’t work. The reasonable thing just doesn’t work.” We are left with the question of what constraints of mutual consistency, if any, one *can* impose on a collection of density matrices that encapsulate the knowledge available to Alice, Bob, Charles, Doris, Edward, . . . about one and the same physical system. I do not know the answer. What I can show, at a minimum, is that Peierls’ second condition remains a valid one, even though he justified it only under the assumption that his primary condition held.

Peierls’ second condition is the concise mathematical expression of the fact that whatever other mutual consistency conditions one might argue for or against, there is one rock-bottom requirement. The different probabilities their density matrices ρ_a and ρ_b enable Alice and Bob to assign to the outcome of any subsequent measurement made on one and the same individual physical system cannot contradict one another. This is a very weak restriction, since neither of their probability assignments can be refuted by the subsequent occurrence or nonoccurrence of any measurement outcome they both assign a probability that is neither 1 nor 0. The knowledge one of them has can contradict the knowledge possessed by the other only if one of them knows that something will certainly happen, while the other knows that it certainly will not. So a minimalist consistency condition is that there can be no outcome of any measurement to which one of them assigns a probability 1 and the other assigns a probability 0. This consistency condition turns out to be

equivalent to Peierls' condition

$$\rho_a \rho_b \neq 0. \quad (9)$$

To see this note first that if (9) fails to hold, so that $\rho_a \rho_b = 0$, then since density matrices are hermitian we also have $\rho_b \rho_a = 0$. So ρ_a and ρ_b commute, and there is a basis of joint eigenstates. Since the vanishing of $\rho_a \rho_b$ requires at least one of ρ_a and ρ_b to have zero eigenvalue for each eigenstate, we can resolve the identity (not necessarily uniquely) into the sum of two projections,

$$1 = P_a + P_b, \quad (10)$$

where P_a and P_b project onto subspaces of eigenstates of ρ_a and ρ_b with eigenvalue zero. The outcomes of any measurement that discriminates between these two orthogonal subspaces will be assigned probabilities 1 or 0 by Alice, and 0 or 1 by Bob. Whatever Alice says must happen Bob will say cannot happen, and vice versa.

The converse is slightly more subtle. Suppose there is some particular measurement with an outcome that Alice gives probability 0 and Bob gives probability 1. This means there is some hermitian operator¹ M with eigenvalues m in the range

$$0 \leq m \leq 1 \quad (11)$$

with

$$\text{Tr} \rho_a M = 0, \quad \text{Tr} \rho_b M = 1. \quad (12)$$

If the eigenstates of M are $|\psi_j\rangle$ with eigenvalues m_j then expanding (12) in the basis of those eigenstates gives

$$\sum_j \langle \psi_j | \rho_a | \psi_j \rangle m_j = 0, \quad (13)$$

and

$$\sum_j \langle \psi_j | \rho_b | \psi_j \rangle m_j = 1, \quad (14)$$

or, since $\text{Tr} \rho_b = 1$,

$$\sum_j \langle \psi_j | \rho_b | \psi_j \rangle (1 - m_j) = 0. \quad (15)$$

¹ If you take an old-fashioned view of what constitutes a measurement it suffices to take the hermitian operator to be a projection operator — i.e. every eigenvalue m is either 0 or 1. If you prefer to think of measurements in terms of positive-operator-valued measures (POVMs), there is no reason to make that restriction. The argument that follows works for either reading of “any measurement”. It thereby demonstrates that the existence of any such generalized measurement implies the existence of an ordinary von Neumann measurement.

Since the diagonal matrix elements of ρ_a and ρ_b are non-negative and since the m_j satisfy (11), we conclude from (13) and (15) that

$$\langle \psi_j | \rho_a | \psi_j \rangle = 0, \quad m_j \neq 0, \quad (16)$$

$$\langle \psi_j | \rho_b | \psi_j \rangle = 0, \quad m_j \neq 1. \quad (17)$$

But if

$$\langle \psi | \rho | \psi \rangle = 0 \quad (18)$$

for a density matrix ρ , then we must have²

$$\rho | \psi \rangle = 0. \quad (19)$$

Therefore (16) and (17) require every eigenstate $|\psi_k\rangle$ of M to be an eigenstate with zero eigenvalue of either ρ_a or ρ_b . It follows that

$$\langle \psi_i | \rho_a \rho_b | \psi_j \rangle = \sum_k \langle \psi_i | \rho_a | \psi_k \rangle \langle \psi_k | \rho_b | \psi_j \rangle = 0 \quad (20)$$

for arbitrary i and j . Since the $|\psi_i\rangle$ are a complete set, we then have

$$\rho_a \rho_b = 0. \quad (21)$$

The existence of any measurement whatever with an outcome that Alice's density matrix requires and Bob's forbids (or vice versa) means that the product of their density matrices must vanish.

So Bob's knowledge can contradict Alice's if and only if their density matrices violate Peierls' second condition (9). This minimalist constraint (9) clearly generalizes to any number of density matrices $\rho_a, \rho_b, \rho_c \dots$ used by Alice, Bob, Carol, ... to encapsulate what each of them knows about one and the same system. No two of them can assign probabilities of 1 and 0 to the outcome of any measurement, and therefore all possible pairs of density matrices must have a non-zero product.

I do not know if one can impose any conditions beyond requiring all pairs of density matrices to satisfy the minimalist condition (9). I have the feeling that if quantum mechanics is really about knowledge and only knowledge, then there ought to be further elementary constraints on the possible density matrices describing one and the same physical system that are stronger than the very weak second condition of Peierls, but not as

² For a density matrix ρ has a hermitian square root, $\sqrt{\rho}$. Since (18) requires the norm of $\sqrt{\rho}|\psi\rangle$ to vanish, so must $\sqrt{\rho}|\psi\rangle$ itself. So $\rho|\psi\rangle = \sqrt{\rho}\sqrt{\rho}|\psi\rangle = 0$.

strong as his overly restrictive first condition. I wish somebody would tell me what they are, or provide me with a convincing argument that there are none.

And I wish it were still possible to talk about these things with John Bell.

Acknowledgment. For some time I went around like Coleridge’s Ancient Mariner, cornering people and asking if they could explain to me Peierls’ first criterion. It was Chris Fuchs who finally pointed out to me that the criterion obviously fails if Alice knows the pure state of a qubit but Bob knows only that it is one or the other of two nonorthogonal states. A remark by Ben Schumacher turned my attention to Peierls’ second criterion, and I have had a stimulating correspondence about it with Rudiger Schack. This work was supported by the National Science Foundation, Grants PHY9722065 and PHY0098429.

Appendix

Every pure spin- $\frac{1}{2}$ state is a spin-up eigenstate along some direction \mathbf{n} and the projection operator on such an eigenstate is $\frac{1}{2}(1 + \mathbf{n} \cdot \boldsymbol{\sigma})$. So regardless of what observable Alice chooses to measure on the right qubit — i.e. regardless of what unitary transformation she applies before measuring it in the computational basis — the post-measurement mixed-state density matrix for the left qubit can be expressed as a probability-weighted sum of the two possible post-measurement pure state density matrices known to Alice as a result of her measurement of the right qubit:

$$\rho = \frac{p}{2}(1 + \mathbf{n} \cdot \boldsymbol{\sigma}) + \frac{q}{2}(1 + \mathbf{m} \cdot \boldsymbol{\sigma}), \quad (22)$$

for probabilities p and $q = 1 - p$. But the density matrix for the left qubit is unaltered by anything Alice does to the right qubit. It must therefore continue to be the reduced density matrix (2), which can be rewritten as

$$\rho = \frac{1}{2}(1 + \cos 2\theta \mathbf{z} \cdot \boldsymbol{\sigma}). \quad (23)$$

Thus p , \mathbf{m} , and \mathbf{n} are subject to the constraint that

$$\mathbf{z} = \sec 2\theta (p\mathbf{n} + q\mathbf{m}). \quad (24)$$

So the unique direction orthogonal to both \mathbf{n} and \mathbf{m} , along which quantum mechanics forbids Bob to have any knowledge of the spin regardless of the outcome of Alice’s measurement, is necessarily orthogonal to \mathbf{z} . Bob’s density matrix (2) does indeed afford him no knowledge whatever of the spin along that one special direction.

References

1. John Bell, “Against Measurement”, *Physics World*, August 1990, 33-40.
2. N. David Mermin, *Space and Time in Special Relativity*, McGraw Hill, New York, 1968; reissued by Waveland Press, Prospect Heights, Illinois, 1989.
3. J. S. Bell, “How to teach special relativity”, *Progress in Scientific Culture* **1**, No 2, summer 1976; reprinted in *Speakable and unspeakable in quantum mechanics*, Cambridge University Press, Cambridge, 1987, 67-80.
4. N. David Mermin, *Phys. Rev. Lett.* **65**, 1838-40 (1990).
5. R. E. Peierls, “In defense of ‘measurement’”, *Physics World*, January 1991, 19-21.
6. Rudolf Peierls, *More Surprises in Theoretical Physics*, Princeton University Press, Princeton, New Jersey, 1991, 11.
7. Quoted by Jeremy Bernstein in *Quantum Profiles*, Princeton University Press, Princeton, New Jersey, 1991, 84.